

## INDEX OF INEQUALITY-ADJUSTED HAPPINESS (IAH) IMPROVED

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### **Abstract**

*Seven years ago we proposed a new measure of happiness in nations, called Inequality-Adjusted Happiness, abbreviated as IAH. This measure indicates how successful nations are in combining a high level of happiness and a low degree of inequality in happiness among citizens. The index gives equal weight to the level of happiness measured using the mean and inequality of happiness measured using the standard deviation. Scores on that index are now available for 139 nations. In this paper we present a technical improvement for calculating IAH using the simple formula  $IAH = 8.28(m - s) + 17.2$*

### **Dual policy aims**

Happiness is a rising topic on the political agenda and this brings back a classic question on priorities in the promotion of happiness. Should we focus on achieving a higher *level* of happiness, or reduce *inequality* in happiness in the first place?.

Both these aims gained prominence in the Enlightenment, when we came to realize that happiness is possible during earthly life and that social inequality is not immutable or divine order. Awareness of this possibility fueled notions of desirability. In the case of happiness this was crystallized in the 'utilitarian' moral philosophy, summarized in Bentham's (1789) credo that we should aim at "the greatest happiness for the greatest number". The desire for more equality gave rise to political philosophies such as socialism. Both ideologies call for indicators of progress in meeting the desired ends.

Progress in creating greater happiness is typically measured using survey studies, in which a representative sample of the general public answers a question such as 'Taking all together, how satisfied are you with your life as-a-whole these days? Please respond by choosing a number between 0 and 10. The happiness of the 'greatest number' is then expressed by the *mean*. Research shows that average happiness has increased over the last decennium in most countries (Veenhoven 2009). Likewise progress in reducing inequality can be measured using the *standard deviation* of responses to survey questions about happiness. Research shows that disparity in happiness has decreased substantially during the last decennium in modern nations (Veenhoven 2005).

## Need for inclusive monitoring

Achievement of one of these aims may come at the expense of the other, for instance when greater happiness of the majority is brought about by exploitation of a minority, or when greater equality results in unhappiness for all, as has happened in former communist countries. Though current social policies appear to add both to a higher level of happiness and to smaller inequality in happiness (Ott 2005), it is worth keeping an eye on the combination. This is common practice in policy making where typically multiple goals are involved. In this case it calls for an indicator that combines both the aim of a higher level of happiness and that of a more equitable distribution of happiness.

## Inequality Adjusted Happiness

In an earlier paper in this journal (Veenhoven & Kalmijn, 2005) we presented such an indicator, which we called the *Inequality-Adjusted Happiness* index, short "IAH". It is a linear combination of the average happiness value  $m$  and the within-nation standard deviation  $s$ . IAH is expressed as a number ranging from 0 to 100.

*Figure 1 about here*

Each nation can be represented in Fig. 1 as a point with abscissa  $m$  and ordinate  $s$ . The average value  $m$  is bounded between the two terminal values of the scale of measurement, represented by the points L and H respectively. It can be proven (Kalmijn, 2010) that for each value of  $m$  the standard deviation  $s$  has a maximum possible value and that only points within or on the circumference of the semicircle with diameter LH and centre M can represent the happiness situation in a nation.

The value 100 corresponds to the most happy situation, which is characterized by the highest possible scale value of  $m$  and  $s=0$ , the point H in Fig. 1. The situation  $IAH = 0$  is a compromise between both views of the worst conceivable situation, which is L according to the utilitarians and T in the extremely egalitarian view. This compromise is represented by a point W on the semicircle circumference somewhere between L and T. Its exact position depends on the weights assigned to both views: the more weight is given to the utilitarian view, the closer W is to L. The *IAH*-value of a nation represented by a point N with  $m$  and  $s$  as its Cartesian coordinates, is obtained by projecting the point N onto the line segment WH, which acts as the '*IAH* axis'. The exact position of the projection depends on the type of projection.

Two options are considered in the 2005 paper, central projection and orthogonal projection, resulting in the projections C and P respectively of N, and the final choice is made in favor of the orthogonal projection.

## *Data on IAH in nations*

Using this approach *IAH* scores have been computed for most nations of

the world. These data are presented in the World Database of Happiness (Veenhoven 2011) in two reports that are continuously updated. In the first report a rank list of current *IAH* in 139 nations is presented (Veenhoven 2011a). On this list Costa Rica is currently at the top of the rank with an *IAH* score of 79 and Tanzania at the bottom with a score of 19. In the second report findings on change in *IAH* since 1973 in 14 western nations are presented, with a rise in 10 nations and a decline in four (Veenhoven 2011b).

### **The problem**

A problem arises with nations that might be represented by a point within the semicircle segment *LW*, because their orthogonal projection falls on the line segment *VW* outside the semicircle; there would have been a similar problem had the central projection had been adopted. The *IAH*-value of these nations is less than that of the compromise situation *W*. So to avoid negative *IAH*-values, we decided to assign the value *IAH*=0 to a nation projected onto *V* and to accept that the *IAH*-value of *W* was a small positive number. In this way-out, the *IAH*-value of the nation *N* is defined as  $100 \times \frac{VP}{VH}$ , and rounded to integer values.

Although the practical consequences of the above decisions seem to be rather modest, at least in the 'equal weights case', the solution described above was felt to be unsatisfactory, but acceptable as long as no better alternative was available.

### **Solution: Skew projection**

In a recent discussion on this statistic, Dr. Jan Koster suggested using a skew projection as an alternative that enables one to circumvent the above problem. In the suggested method, the projector of *N* is drawn parallel to the tangent *WZ* in *W* to the semicircle. The point of intersection *U* of the line segment *WH* and this projector through *N* defines the 'improved' *IAH*-value of the nation *N* as  $100 \times \frac{WU}{WH}$ , and rounded to integer values. In this way, projection outside the semicircle is virtually impossible.

#### *Consequences of the application of the skew projection method*

The most important consequence is that the point *W* has the zero *IAH*-value it should have and all other *IAH*-values are positive. The improved *IAH*-values can be calculated using a formula that is derived in Appendix A. In the case equal weights are assigned to both the utilitarian and the egalitarian view, and happiness is quantified on a 0 to 10 range, the *IAH*-value can be simply calculated as

$$IAH = 8.28 (m - s) + 17.2$$

The improved *IAH*-values in 139 nations are listed in Appendix B together with the original scores. Their numerical values are slightly smaller than the original; for most nations the magnitude of the difference is of the

order of 4 units. Some Latin-American nations with relatively high values of both  $m$  and  $s$  show a somewhat stronger reduction. The new values give a more valid indication of the extent to which the happiness situation in a nation is remote from the 'ideal' situation represented by the point H.

## Conclusion

The application of a skew projection results in Inequality-Adjusted Happiness values with properties superior to those of the originally defined index and we recommend that these are replaced using the *IAH* skew projection described in this paper.

## Acknowledgement

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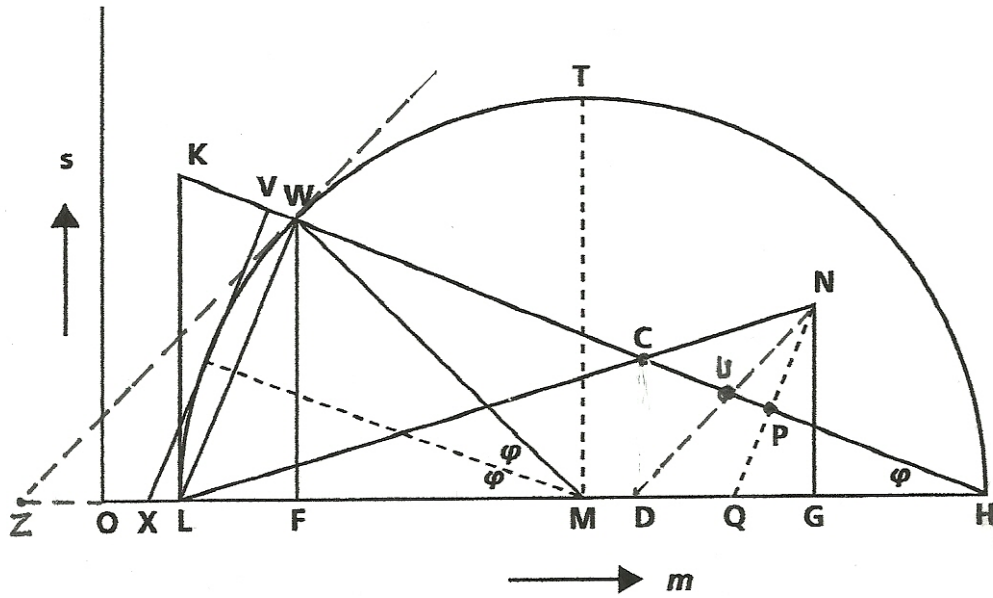


Figure 1. Three projections of  $N$  on the  $IAH$ -axis  $WH$ : central ( $C$ ), orthogonal ( $P$ ) and skew ( $U$ ).

## Appendix A

### Improved IAH by the application of skew projection

Let  $M$  be the centre of the semicircle with radius  $r$ . A nation can be represented by its mean happiness value  $m = \underline{LG}$  as its abscissa and its internal standard deviation  $s = \underline{NG}$  as its ordinate.

Let  $W$  represent the compromise for the worst conceivable situation and  $WZ$  be the tangent through  $W$  to the semicircle. The skew projection of  $N$  onto the IAH-axis  $WH$  is obtained as the intersection of  $WH$  with the line segment  $ND$  through  $N$  parallel to  $WZ$ . In that case, the IAH-value of  $N$  equals the ratio  $(\underline{UW} / \underline{HW}) \times 100$ .

From the parallelism of  $WZ$  and  $UD$  follows the proportionality

$$\underline{UW} / \underline{HW} = \underline{DZ} / \underline{HZ}.$$

The angles  $ZMW$  and  $DNG$  are equal; let their value be  $2J$ , where  $J = \text{angle}(WHL)$ . The value of  $2J$  equals  $[w_E / (w_E + w_U)](\pi/2)$ , where  $w_E$  and  $w_U$  are the weights assigned to the egalitarian and utilitarian views respectively. Since

$$\begin{aligned} \underline{ZD} &= \underline{ZG} - \underline{DG} = \underline{ZL} + \underline{LG} - \underline{DG} = \underline{ZM} - \underline{ML} + \underline{LG} - \underline{DG} \\ &= r/\cos 2J - r + m - s \tan 2J, \text{ and} \end{aligned}$$

$$\underline{ZH} = \underline{ZM} + \underline{MH} = r/\cos 2J + r,$$

the IAH-value of the nation represented by the point  $N(m, s)$  equals

$$\frac{\frac{r}{\cos 2\varphi} - r + m - s \tan 2\varphi}{\frac{r}{\cos 2\varphi} + r} \times 100$$

This result can also be written as

$$IAH = \frac{(m - r) \cos 2\varphi + r - s \sin 2\varphi}{r + r \cos 2\varphi} \times 100$$

In the case of equal weights  $w_E = w_U$  and  $2\varphi = \pi/4$ ; when happiness is quantified on a  $[0, 10]$  scale, then  $r = 5$  and in this particular case this formula can be simplified to  $IAH = 8.28(m - s) + 17.2$ .

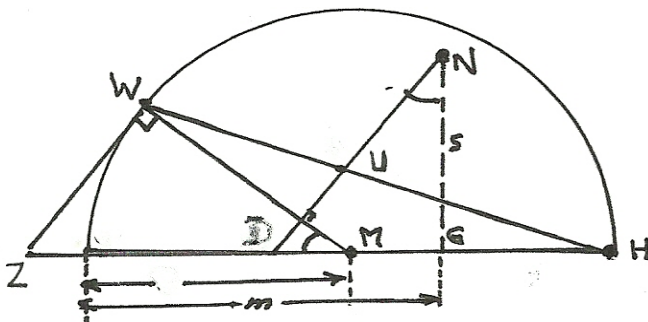


Figure 2. Skew projection U of the nation N onto the IAH-axis WH.

## Appendix B

**Inequality Adjusted Happiness (IAH) in 135 nations 2000-2009**

Scores using original and improved version

| Nation              | Happiness   |                           | Inequality adjusted happiness |                     |
|---------------------|-------------|---------------------------|-------------------------------|---------------------|
|                     | <i>Mean</i> | <i>Standard deviation</i> | <i>IAH original</i>           | <i>IAH improved</i> |
| Afghanistan         | 4,1         | 2,04                      | 36                            | 35                  |
| Albania             | 4,6         | 2,5                       | 38                            | 35                  |
| Algeria             | 5,4         | 2,56                      | 46                            | 41                  |
| Andorra             | 6,8         | 1,8                       | 62                            | 59                  |
| Angola              | 4,3         | 3,19                      | 32                            | 26                  |
| Argentina           | 7,3         | 1,97                      | 67                            | 62                  |
| Armenia             | 5           | 2,01                      | 44                            | 42                  |
| Australia           | 7,7         | 1,7                       | 71                            | 67                  |
| Austria             | 7,6         | 1,95                      | 69                            | 64                  |
| Azerbaijan          | 5,3         | 2,06                      | 47                            | 44                  |
| Bangladesh          | 5,3         | 2,26                      | 46                            | 42                  |
| Belarus             | 5,2         | 2,02                      | 46                            | 43                  |
| Belgium             | 7,3         | 1,77                      | 67                            | 63                  |
| Belize              | 6,6         | 2                         | 60                            | 55                  |
| Benin               | 3           | 2,7                       | 22                            | 20                  |
| Bolivia             | 6,5         | 1,83                      | 59                            | 56                  |
| Bosnia Hercegovina  | 5,6         | 2,66                      | 47                            | 41                  |
| Botswana            | 4,7         | 2,42                      | 40                            | 36                  |
| Brazil              | 7,5         | 2,16                      | 67                            | 61                  |
| Bulgaria            | 4,4         | 2,54                      | 36                            | 33                  |
| Burkina Faso        | 4,4         | 2,02                      | 38                            | 37                  |
| Burundi             | 2,9         | 1,96                      | 24                            | 25                  |
| Cambodia            | 4,9         | 2,12                      | 43                            | 40                  |
| Cameroon            | 3,9         | 1,69                      | 35                            | 36                  |
| Canada              | 7,8         | 1,91                      | 71                            | 66                  |
| Central African Rep | 4,6         |                           |                               |                     |
| Chad                | 5,4         | 2,61                      | 45                            | 40                  |
| Chile               | 6,6         | 2,29                      | 59                            | 53                  |
| China               | 6,3         | 2,71                      | 54                            | 47                  |
| Colombia            | 7,7         | 2,14                      | 70                            | 63                  |
| Congo (Brazzaville) | 3,7         | 2,93                      | 27                            | 23                  |
| Congo (Kinshasa),   | 4,4         |                           |                               |                     |
| Costa Rica          | 8,5         | 1,71                      | 79                            | 73                  |

| Nation             | Happiness   |                           | Inequality adjusted happiness |                     |
|--------------------|-------------|---------------------------|-------------------------------|---------------------|
|                    | <i>Mean</i> | <i>Standard deviation</i> | <i>IAH original</i>           | <i>IAH improved</i> |
| Cote d'Ivoire      | 4,4         | 1,63                      | 40                            | 41                  |
| Croatia            | 6           | 2,48                      | 52                            | 47                  |
| Cyprus             | 7,1         | 2,04                      | 64                            | 59                  |
| Czech Republic     | 6,5         | 2,18                      | 58                            | 53                  |
| Denmark            | 8,3         | 1,55                      | 78                            | 73                  |
| Djibouti           | 5,7         | 2,54                      | 48                            | 43                  |
| Dominican Republic | 7,6         | 2,31                      | 68                            | 61                  |
| Ecuador            | 6,4         | 2,04                      | 57                            | 53                  |
| Egypt              | 5,7         | 2,89                      | 47                            | 40                  |
| El Salvador        | 6,7         | 2,02                      | 60                            | 56                  |
| Estonia            | 6           | 2,14                      | 53                            | 49                  |
| Ethiopia           | 4,2         | 2,25                      | 35                            | 33                  |
| Finland            | 7,9         | 1,61                      | 73                            | 69                  |
| France             | 6,6         | 2,16                      | 59                            | 54                  |
| Georgia            | 4,3         | 1,91                      | 37                            | 37                  |
| Germany            | 7,1         | 2                         | 64                            | 60                  |
| Ghana              | 5,2         | 2,35                      | 45                            | 41                  |
| Greece             | 6,4         | 2,23                      | 57                            | 52                  |
| Guatemala          | 7,2         |                           |                               |                     |
| Guinea             | 4,5         |                           |                               |                     |
| Guyana             | 6,5         | 1,97                      | 59                            | 55                  |
| Haiti              | 3,9         |                           |                               |                     |
| Honduras           | 7           | 2,45                      | 62                            | 55                  |
| Hong Kong          | 6           | 2,14                      | 53                            | 49                  |
| Hungary            | 5,5         | 2,48                      | 47                            | 42                  |
| Iceland            | 8,2         | 1,63                      | 76                            | 71                  |
| India              | 5,5         | 2,13                      | 48                            | 45                  |
| Indonesia          | 6,3         | 2,19                      | 56                            | 51                  |
| Iran               | 5,9         | 2,55                      | 50                            | 45                  |
| Iraq               | 4,7         | 2,45                      | 39                            | 36                  |
| Ireland            | 7,6         | 1,84                      | 70                            | 65                  |
| Israel             | 7           | 2,29                      | 62                            | 56                  |
| Italy              | 6,7         | 1,9                       | 61                            | 57                  |
| Jamaica            | 6,7         |                           |                               |                     |
| Japan              | 6,5         | 2,03                      | 58                            | 54                  |
| Jordan             | 5,9         | 2,51                      | 51                            | 45                  |

| Nation         | Happiness   |                           | Inequality adjusted happiness |                     |
|----------------|-------------|---------------------------|-------------------------------|---------------------|
|                | <i>Mean</i> | <i>Standard deviation</i> | <i>IAH original</i>           | <i>IAH improved</i> |
| Kazakhstan     | 6,1         | 1,9                       | 55                            | 52                  |
| Kenya          | 3,7         | 2,14                      | 31                            | 30                  |
| Korea, (South) | 6           | 2,03                      | 54                            | 50                  |
| Kosovo         | 5,4         | 2,19                      | 47                            | 43                  |
| Kuwait         | 6,6         |                           |                               |                     |
| Kyrgyzstan     | 5,5         | 2,32                      | 48                            | 44                  |
| Laos           | 6,2         | 1,72                      | 57                            | 55                  |
| Latvia         | 5,4         | 2,19                      | 47                            | 43                  |
| Lebanon        | 4,7         | 2,39                      | 40                            | 36                  |
| Liberia        | 4,3         |                           |                               |                     |
| Lithuania      | 5,5         | 2,24                      | 48                            | 44                  |
| Luxembourg     | 7,7         | 1,96                      | 70                            | 64                  |
| Macedonia      | 4,7         | 2,84                      | 37                            | 32                  |
| Madagascar     | 3,7         | 1,86                      | 32                            | 33                  |
| Malawi         | 6,2         |                           |                               |                     |
| Malaysia       | 6,5         | 1,84                      | 60                            | 56                  |
| Mali           | 4,7         | 2,9                       | 38                            | 32                  |
| Malta          | 7,1         | 2,15                      | 64                            | 58                  |
| Mauritania     | 4,9         | 2,62                      | 41                            | 36                  |
| Mexico         | 7,9         | 2,22                      | 71                            | 64                  |
| Moldova        | 4,9         | 2,36                      | 41                            | 38                  |
| Mongolia       | 5,7         | 2,37                      | 49                            | 44                  |
| Montenegro     | 5,2         | 2,64                      | 43                            | 38                  |
| Morocco        | 5,4         | 2,22                      | 47                            | 43                  |
| Mozambique     | 3,8         | 1,76                      | 34                            | 34                  |
| Namibia        | 5,2         |                           |                               |                     |
| Nepal          | 5,3         | 1,65                      | 49                            | 48                  |
| Netherlands    | 7,6         | 1,42                      | 71                            | 68                  |
| New Zealand    | 7,5         | 1,94                      | 68                            | 63                  |
| Nicaragua      | 7,1         | 2,5                       | 62                            | 55                  |
| Niger          | 3,8         | 1,75                      | 33                            | 34                  |
| Nigeria        | 5,7         | 2,16                      | 50                            | 46                  |
| Norway         | 7,9         | 1,67                      | 73                            | 68                  |
| Pakistan       | 5           | 2,49                      | 42                            | 38                  |
| Palestina      | 4,9         | 2,42                      | 42                            | 38                  |
| Panama         | 7,8         | 2,01                      | 71                            | 65                  |

| Nation               | Happiness   |                           | Inequality adjusted happiness |                     |
|----------------------|-------------|---------------------------|-------------------------------|---------------------|
|                      | <i>Mean</i> | <i>Standard deviation</i> | <i>IAH original</i>           | <i>IAH improved</i> |
| Paraguay             | 6,9         | 2,04                      | 62                            | 57                  |
| Peru                 | 6,2         | 2,37                      | 54                            | 49                  |
| Philippines          | 5,9         | 2,49                      | 51                            | 45                  |
| Poland               | 6,4         | 2,36                      | 56                            | 51                  |
| Portugal             | 5,7         | 2,16                      | 50                            | 47                  |
| Qatar                | 6,8         | 1,89                      | 61                            | 57                  |
| Romania              | 5,7         | 2,47                      | 49                            | 44                  |
| Russia               | 5,5         | 2,46                      | 47                            | 43                  |
| Rwanda               | 4,3         | 2,33                      | 36                            | 34                  |
| Saudi Arabia         | 6,5         | 2,13                      | 58                            | 53                  |
| Senegal              | 4,5         | 1,62                      | 41                            | 41                  |
| Serbia               | 5,4         | 2,53                      | 46                            | 41                  |
| Sierra Leone         | 3,5         | 2,36                      | 29                            | 27                  |
| Singapore            | 6,9         | 1,73                      | 63                            | 60                  |
| Slovakia             | 5,9         | 2,39                      | 51                            | 46                  |
| Slovenia             | 6,9         | 2,16                      | 61                            | 56                  |
| South Africa         | 5,8         | 2,69                      | 49                            | 43                  |
| Spain                | 7,2         | 1,81                      | 66                            | 62                  |
| Sri Lanka            | 5,1         | 2,07                      | 44                            | 42                  |
| Sudan                | 5           |                           |                               |                     |
| Sweden               | 7,8         | 1,72                      | 72                            | 68                  |
| Switzerland          | 8           | 1,55                      | 75                            | 71                  |
| Syria                | 5,9         | 1,99                      | 53                            | 50                  |
| Taiwan               | 6,2         | 2,29                      | 54                            | 50                  |
| Tajikistan           | 5,1         | 1,68                      | 46                            | 46                  |
| Tanzania             | 2,8         | 2,92                      | 19                            | 16                  |
| Thailand             | 6,6         | 1,82                      | 60                            | 57                  |
| Togo                 | 2,6         | 2,08                      | 21                            | 22                  |
| Trinidad en Tobago   | 7           | 2,4                       | 62                            | 55                  |
| Tunisia              | 5,9         | 1,86                      | 53                            | 51                  |
| Turkey               | 5,6         | 2,62                      | 47                            | 42                  |
| Turkmenistan         | 7,2         |                           |                               |                     |
| Uganda               | 4,8         | 2,32                      | 41                            | 38                  |
| Ukraine              | 5           | 2,34                      | 42                            | 39                  |
| United Arab Emirates | 7,3         |                           |                               |                     |

| Nation         | Happiness   |                           | Inequality adjusted happiness |                     |
|----------------|-------------|---------------------------|-------------------------------|---------------------|
|                | <i>Mean</i> | <i>Standard deviation</i> | <i>IAH original</i>           | <i>IAH improved</i> |
| United Kingdom | 7,1         | 2                         | 65                            | 60                  |
| United States  | 7,4         | 1,85                      | 68                            | 63                  |
| Uruguay        | 6,8         | 1,99                      | 61                            | 57                  |
| Uzbekistan     | 6           | 1,96                      | 54                            | 51                  |
| Venezuela      | 7,5         | 2,77                      | 65                            | 57                  |
| Viet Nam       | 6,1         | 1,94                      | 55                            | 52                  |
| Yemen          | 4,8         |                           |                               |                     |
| Zambia         | 5           | 2,61                      | 41                            | 37                  |
| Zimbabwe       | 3           | 2,42                      | 24                            | 22                  |